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Brief Communication

Comments on the unification of electromagnetism and gravitation through "Generalized Einstein manifolds"

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Abstract

Akbar-Zadeh [J. Geom. Phys. 17 (1995) 342] has recently proposed a new geometric formulation of Einstein–Maxwell system *with source* in terms of what are called "Generalized Einstein manifolds". We show that, contrary to the claim, Maxwell equations have not been derived in this formulation, and that the assumed equations can be identified only as *source-free* Maxwell equations in the proposed geometric setup. A genuine derivation of source-free Maxwell equations is presented within the same framework. We draw a conclusion that the proposed unification scheme can pertain only to source-free situations. © 1998 Elsevier Science B.V.

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In a recent article [1], using the tangent bundle approach to Finsler geometry, Akbar-Zadeh has introduced a class of Finslerian manifolds called "Generalized Einstein manifolds". These manifolds are obtained through some constrained metric variations on an action functional depending on the curvature tensors. The author has then proposed a new scheme for the unification of electromagnetism and gravitation, in which the space–time manifold, M, with its usual pseudo-Riemannian metric, $g_{ij}(x)$, is endowed with a Finslerian connection containing the Maxwell tensor, $F^{ij}(x)$. Following this scheme, the author arrives at a class of Generalized Einstein manifolds containing the solutions of Einstein–Maxwell equations. As for Maxwell equations, they are declared [1, pp. 343, 378] to have been obtained by means of Bianchi identities. We wish to point out the following flaws in the treatment of Einstein–Maxwell system.

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First consider the treatment of Maxwell equations. Through some constrained metric variations, and the use of Bianchi identities, the author arrives at [1, Eq. (5.55)]:

$$\nabla_i F^{ij} = \mu_1 u^j, \tag{1}$$

where μ_1 and u^j are defined by [1, Eqs. (5.14) and (2.7)]:

$$\mu_1 = -u^r \nabla_i F_r{}^i, \tag{2}$$

$$u^r = \frac{v^r}{F}. \tag{3}$$

Using notations of [1] throughout, v^r are fiber coordinates of the tangent bundle over M and ∇_i is the usual Riemannian covariant derivative defined through $g_{ij}(x)$. *Assuming* that μ_1 is the proper charge density [1, p. 378], the author then identifies (1) as the Maxwell equations *with source.* The author has, therefore, assumed that

$$\mu_1 = \mu_1(x). \tag{4}$$

However, this *assumption*, together with *definition* (2), already implies Eq. (1). To see this, differentiate (2) with respect to v^j and then use (4) to obtain

$$\nabla_i F_j{}^i = u^r \frac{\partial F}{\partial v^j} \nabla_i F_r{}^i;$$

noting that $\partial F/\partial v^j = u_j$, and using (2) again, we arrive at (1). Therefore, rather than being derived, (1) has in fact been merely assumed.

More importantly, assumption (4) implies that $\mu_1 = 0$, so that *the assumed equations can be identified only as source-free Maxwell equations.* To see this simply differentiate (1) with respect to v^k to obtain

$$0 = \mu_1(\delta^j{}_k - u^j u_k).$$

To clarify this apparently curious result, we note that in the tangent bundle picture of Finsler geometry, fiber coordinates v^k are parameters independent of x. However, for a system of charged particles, for which we can write Maxwell equations, the velocity vector is a function of x. Therefore (1) *cannot* be identified as Maxwell equations *with source* because u^j in this equation are independent of x and (contrary to [1, p. 378]) cannot be considered as a velocity field.

There is, in fact, a genuine derivation of source-free Maxwell equations within the same framework. From the connection given in [1, Eqs. (5.1)–(5.3)] we can directly calculate H, the second contraction of the Berwald curvature, $H^i{}_{jkl}$, in tems of the Riemannian scalar curvature, R, and the Maxwell tensor:

$$H = R + \tfrac{3}{2}K^2 F_{rs}F^{rs} - \tfrac{5}{2}K\mu_1. \tag{5}$$

Eq. (5) is the same as Eq. (5.62) of [1], however, in derivation of the latter, Maxwell equations have been used. This is unnecessary and could have been easily avoided if, in its derivation, Eq. (5.58) was used in place of Eq. (5.59). Now since $H = H(x)$ from the metric variations (see [1, p. 378]), we conclude from (5) that μ_1 is a function of x only. Therefore, we have *derived* (4), and as explained before, this easily leads to the source-free Maxwell equations.

Coming to the treatment of Einstein–Maxwell equations in [1], we note that, because the mass density, ρ_1, has been curiously assumed [1, p. 379] to be proportional to μ_1; [1] it, too, must be zero. Consequently the proposed geometric formulation of Einstein–Maxwell system can pertain only to source-free situations.

References

[1] H. Akbar-Zadeh, Generalized Einstein manifolds, J. Geom. Phys. 17 (1995) 342.

[1] The charge-to-mass ratio of the particles is, therefore, implicitly assumed to be the same.